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ABSTRACT

Numerous research studies over the past five years indicate that a high percentage of college students have severe deficiencies in their ability to use mathematics as a language for representing simple quantitative relationships. In the United States these problems are nearly as common among students with strong computational skills in algebra and calculus as among those with very limited mathematical preparations. Several specific difficulties seem to stem from differences between the structure of mathematics and that of English. Data are presented from a quiz admininstered in four countries: Japan, Israel, Fiji, and the United States. Additional information is provided on China, Great Britain, and Nigeria. Although Japanese and Chinese students far out perform those from other countries there are certain common features across all groups which suggest that the oriental strength stems from education rather than culture or language. (Author)

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REPRESENTATION OF MATHEMATICAL RELATIONSHIPS IN FOUR COUNTRIES:

A Study of College Students' Mathematical Fluency

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REPRESENTATION OF MATHEMATICAL RELATIONSHIPS IN FOUR COUNTRIES:

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Abstract

Studies by Clement, Lochhead, Monk (1981), Ehrlich, Soloway, Abbott, (1983), and Kaput and Sims-Knight (1983) have shown that in the United States a very high percentage of mathematically trained college students lack basic fluency in the use of mathematics as a language for representing quantitative relationships. Most errors made by these students stem from an incorrect model for how mathematical expressions code information; this model seems to have been borrowed from english where it is usually appropriate. We began our study in an attempt to answer two questions raised by the previous data. First, are such representational-skills important in mathematical applications? Second, are the incorrect models used by students peculiar to english speakers? Our goal was not to get a definitive answer to either question but rather to explore whether cross cultural data could be useful to such an investigation.

We felt that data from Israel and Japan might help to answer the first question. These countries have high standards for technical training yet they stress rote learning to a greater extent than is common in the United States. If representational skills are unimportant to applications, then there would be little reason to expect them to be any stronger in Japan and Israel then in the United States. This, however, is not what we found. The issue of language appears to be quite complicated. However, there was a tendency to make errors in all the cultural and language groups we investigated. The similar nature of these errors suggests that they stem from fundamental aspects of human cognition rather than confusions peculiar to a specific language group. Implication for mathematics instruction will be considered.

Abstract

Numerous research studies over the past five years indicate that a high percentage of college students have severe deficiencies in their ability to use mathematics as a language for representing simple quantitative relationships. In the United States these problems are nearly as common among students with strong computational skills in algebra and calculus as among those with very limited mathematical preparations. Several specific difficulties seem to stem from differences between the structure of mathematics and that of english. We investigated the relative frequency of these errors across culture and language. Data is presented from four countries: Japan, Israel, Figi, and the United States. Additional information is provided on China, Great Britain, and Nigeria. Although Japanese and Chinese students far out perform those from other countries, there are certain common features across all groups which suggest that the Oriental strength stems from education rather than culture or language.

Translation Errors

According to Polya (1962) teaching students to solve word problems is "the most important single task of mathematical instruction." For most students the part that is hardest to learn is the translation between natural language and standard mathematical notation. Polya (1957) provides the following description:

> In order to translate a sentence from English into French two things are necessary; First, we must understand thoroughly the English sentence. Second, we must be familiar with the forms of expression peculiar to the French language. The situation is very similar when we attempt to express in mathematical symbols a condition proposed in words. First, we must understand thoroughly the condition. Second, we must be familiar with the forms of mathematical expression
>
> An English sentence is relatively easy translate into French if it can be translated word for word. But there are English idioms which cannot be translated into French word for word. If our sentence contains such idioms, the translation becomes difficult; we have to pay less attention to the separate words, and more attention to the whole meaning; before translating the sentence, we may have to rearrange it.
>
> Polya, G., How to Solve It, 1957

Although the truth of Polya's statement may seem self-evident, two terms remain ill-defined. They are "understand" and "meaning." While no one has difficulty understanding what these words mean they have so far escaped rigorous psychological definition or an objective criterion for

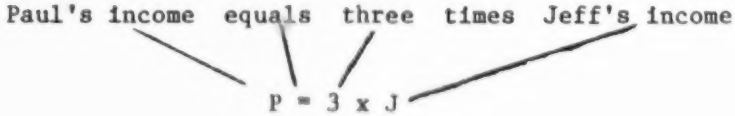
measurement. It is perhaps for that reason that the cognitive psychologists Paige and Simon (1966) offered the following explanation of translation:

> We know, roughly, how to go about translating a statement form English into French: First, the French equivalent of each English word is looked up in a dictionary, and the string of french words substituted for the English string then certain rearrangements (and inflectional modifications) are made in the resulting string so that it will satisfy the rules of French syntax rather than English syntax. Of course this is only a rough paraphrase of how the translation would actually be done, but it indicates the essential elements. It is an example of what we shall call a DIRECT PROCESS.

Paige, J. and H. Simon, Problem Solving Research, Method and Theory, Ed., B. Kleinmuntz, 1966.

This mechanistic approach has the advantage that it is easy to teach. In fact, most algebra text books use some version of the word order matching strategy.

Paul's income equals three times Jeff's income.



$$P = 3 \times J$$

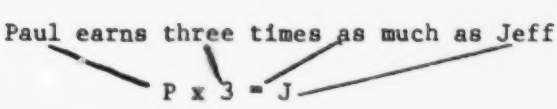
or

$$P = 3J$$

From Developing Mathematical Skills by A. Whimbey and J. Lochhead. P.320

The scheme, however, has one slight disadvantage; it doesn't work.

If for example, the English sentence had read:



The same strategy could produce the reverse relationship. In fact, when the question:

> Write an equation using the variable S and P to represent the following statement: "There are six times as many students as professors at this University." Use S for the number of students and P for the number of professors.

was given to a group of 150 freshmen engineering students (Clement, Lochhead, Soloway, 1979) less than 65% gave a correct answer even though over 90% correctly answered.

> At a Red Sox game there are 3 hotdog sellers for every 2 coke sellers. There are 40 coke sellers in all. How many hotdog sellers are there at the game?

Worse still, less than 30% of this group answered the following problem correctly.

> Write an equation using the variables C and S to represent the following statement: "At Mindy's restaurant, for every four people who ordered cheesecake, there are five people who ordered strudel." Let C represent the number of cheesecakes and S represent the number of strudels ordered."

Contrary to the predictions of many math teachers these errors are not the result of momentary lapses in concentration. Extensive interviews have shown that students find these problems highly confusing. (Clement and Rosnick, 1982)

In fact, Rosnick (1981) found that over 20% of second semester college calculus students refused to accept the correct answer to our simplest example even when it was given to them directly. Rosnick asked

the following question:

> At this university, there are six times as many students as professors. This fact is represented by the equation S = 6P.
>
> A) In this equation, what does the letter P stand for?
>
> I) Professors
>
> II) Professor
>
> III) Number of Professors
>
> IV) None of the above
>
> V) More than one of the above (If so, indicate which ones)
>
> VI) Don't know

B) What does the letter S stand for?

> I) Professor
>
> II) Student
>
> III) Students
>
> IV) Number of Students
>
> V) None of the above
>
> VI) More than one of the above (If so, indicate which ones)
>
> VII) Don't know

and found nearly a quarter of the responses to be A: IV together with B: I!

Origins of Translation Errors

The various causes of these pervasive errors have been studied extensively by Clement, Lochhead, Monk (1981) and Kaput and Sims-Knight (1983) as well as many other investigators. Faulty application of the

word order matching strategy accounts for only some of the errors. Other significant factors include confusion between the conceptual syntax of algebra and that of both English and pictorial representations.

In the equation S = 6P, it is important to interpret the 6 as an operator acting on P to increase the number which P represents. But if the expression is read descriptively, as it normally would be in most natural languages, then the "6" may be interpreted as an adjective modifying P where P is basically a label for "professor". Similarly if the equation is viewed as a pictagraphic representation then we see six professors along side a single student. The importance of this effect is revealed in the very high error rates found in translations from pictures to equations.

In terms of Polya's description of the translation process, errors are being made at the second stage: "must be familiar with the forms of mathematical expression." Students simply don't recognize that the syntax of algebra is different from that of English and from that of pictures. In many cases they are hardly conscious of the role syntax plays in interpretation and thus find the expressions meaning alternaties mysteriously. A mathematics major once described the phenomenon to me as: "When I think of it like English it is 6S=P but when I think of it like math it is S=6P. I am not sure which is right."

Difficulties of this type are of considerable psychological interest. The conflict between alternative interpretations may be related to issues of hemispheric dominance, with the left symbolic hemisphere losing control to the right side in the presence of strong pictorial cues. If something of this sort is involved then it may be possible to learn about the mechanism through studying groups with different forms of hemispheric specialization. Similarly language plays

an important role in these translation difficulties. One might for example, conjecture that a language from pictographic tradition would encourage errors since it would tend to emphasize visual rather than operational interpretations. Before ideas such as these can be investigated, it is first necessary to establish whether there is any evidence that performance on translation tasks is effected by linguistic or cultural factors. Such effects must, unfortunately, be confounded with differences in educational practices.

An International Survey

The international data presented here is based on a simple five item quiz (see figure 1) that takes about 15 minutes to administer. Three questions (1,3,5) emphasize translation; two test calculational fluency (2,4) in the context of word problems. Six different groups were tested (see figure 2) with results shown in figure 3 and table I.

The most obvious conclusion is that performance does vary significantly across language and culture. However, there also are some similarities. Problem 3, the picture to algebra problem, gave all groups some trouble. This suggests that a fundamental characteristic of human cognition interferes with the picture to algebra translation. Nonetheless, the Japanese students out perform all other groups on this question by at least a factor of two. Is this the result of education or of something else? Similarly the Israeli college group performed much worse on this question than on any of the others. Could that be an indication of a cultural or linguistic factor?

Our study was not designed to answer questions, but rather to find out what questions might be worth asking. Nonetheless the overall pattern suggests that education may be the most significant factor influencing performances.

The Test

The five questions in the test were the following:

1. Write an equation using the variables X and Y to represent the following statement: "In one school, there are six times as many students as there are teachers". Use X for the number of students and Y for the number of teachers.

2. A man owns 2 cars and drives one of them to work every day. His first car gets 12 miles per gallon and uses 3 gallons for this trip. His second car gets 18 miles per gallon and uses ? gallons for this trip.

3. A man takes a photograph from an airplane of some of the cows and pigs in a large field full of cows and pigs. He is sure that he has photographed a typical sample of the animals in the field. Write an equation using the letters C and P to describe the relationship between C, the number of cows, and P, the number of pigs in the field. The equation should allow you to calculate the number of cows if given the number of pigs.

4. A store sells 5 bananas for every 7 apples. If one day they sell 35 bananas, how many apples did they sell that day?

5. The equation A=7B gives the relative number of apples, A, and bananas, B, in a bowl of fruit. Does the bowl contain more apples or more bananas?

Figure 1

SUBJECTS

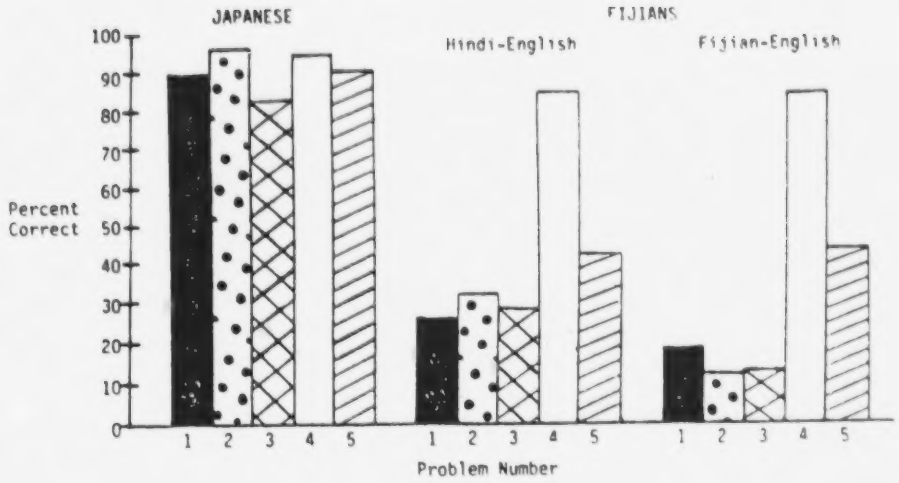
Fijians: Two distinct ethno-linguistic groups of Fijians took part in the study. The first group consisted of 40 female and 39 male Hindi-English bilinguals. The second group consisted of 20 female and 18 male Fijian-English bilinguals. Both groups were second year college students enrolled in the Diploma in Education program. English is the language used in the schools in Fiji, and the test was given to the students in English. All students had taken a math course each year in school prior to entering college.

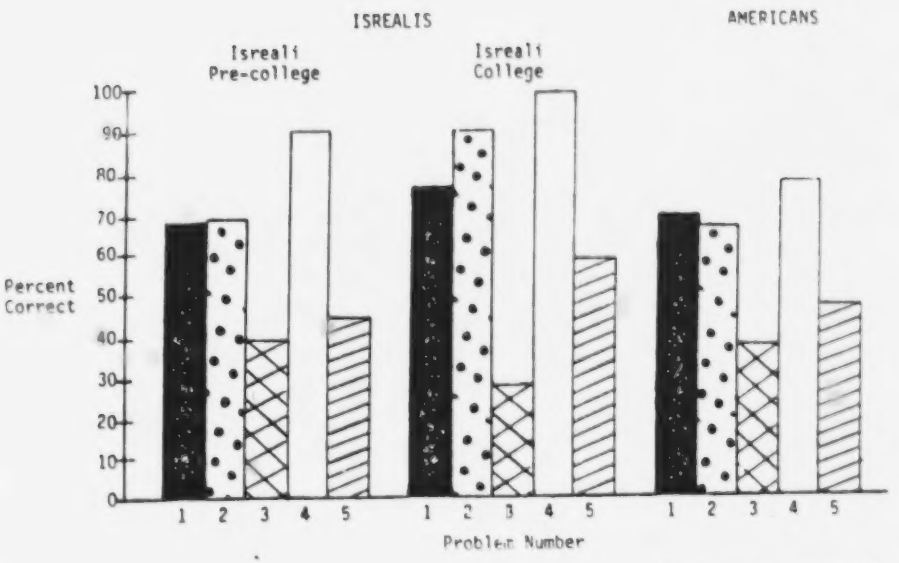
Israeli: The Israeli group consisted of a college and a pre-college group. The college group consisted of 14 female, and 27 male freshman agriculture students who were taking a calculus course. The pre-college group consisted of 60 female and 43 male 7^{th} to 10^{th} graders. Most students were bilingual, with Hebrew being the language understood by all the students. The test was given in Hebrew.

Japanese: The Japanese group consisted of 59 female and 112 male undergraduates enrolled in a social science program (most in sociology or pre-law) in a university in Tokyo. These students had either taken or were currently enrolled in courses in elementary statistics and computer programming. The test was administered in Japanese.

American: The American group consisted of 37 female and 48 male freshman undergraduates enrolled in a math problem-solving course for non-technical majors at the University of Massachusetts. According to a math placement exam, these students were not prepared to enroll in a pre-calculus course.

Figure 2





from Mestre and Lochhead (1983)

Figure 3

Table 1
Breakdown of Answers

Answers Probs. 1-5	Fijians		Israeli		Japanese	American
	Hindu N=79	Native N=38	College N=41	Pre-Coll. N=103	N=171	N=85
X=6Y	25.3	18.4	75.6	67.0	88.3	73.5
6X=Y	62.0	52.6	22.0	32.0	11.7	16.5
6X+Y, 6Y+X, or 6X+Y=1	8.9	26.3	0.	0.	0.	7.1
Other	3.8	2.6	2.4	1.0	0.	4.7
2 gallons	31.6	13.1	90.2	68.0	96.5	67.1
4.5 gallons	49.4	55.3	9.8	25.2	2.3	5.9
6 gallons or 9 gallons	7.6	15.8	0.	0.	0.	8.2
Other	11.4	15.8	0.	6.8	1.2	18.8
C=5P	27.8	13.2	29.3	39.8	82.5	37.6
5C=P	45.6	47.4	61.0	47.6	4.1	24.7
5C+P=6, 5P+C=6, and types □+□=□	19.0	13.2	0.	1.0	3.5	1.2
5C:1P, 5C/1P, or C:P=5:1	3.8	10.5	0.	0.	7.0	8.2
Other	3.8	15.8	9.8	11.6	2.9	28.2
49 Apples	84.8	84.2	100.	90.3	94.2	78.8
37 Apples	3.8	5.3	0.	1.9	0.	2.4
25 Apples	1.3	0.	0.	0.	4.7	2.4
Other	10.1	10.5	0.	7.8	1.2	16.5
More Apples	43.0	44.7	58.5	45.6	90.0	47.1
More Bananas	57.0	55.3	22.0	42.7	8.8	36.5
Equal # of each	0.	0.	4.9	5.8	0.	3.5
Other	0.	0.	14.6	5.8	1.2	12.9

from Mestre and Lochhead

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Results in Fiji, a developing country, were low for both groups. Indians are stereotyped as being good in math; thus if education has little effect on performance we would have expected them to perform relatively high. (A small sample in Nigeria by B. Ojukwu yielded a similar level of performance suggesting a general result for developing countries. The differences between the U.S. and Israel are also in keeping with expectations based on standard stereotypes. Israel is rumored to have a more disciplined education which emphasizes performance in standard problems but probably encourages rote learning more than in the case in the U.S. Thus we would expect higher performance on problem 2 and 4 but equal or lower performance on 1, 3, 5. This is what we found.

Finally there is Japan, (and China since results collected by Herb Simon indicate similar performance there). Their education system is supposed to place stress on rote learning but that may not be entirely fair. Easley and Easley (1982) found that the Japanese, beginning with grade 1, put special emphasis on translations between natural language and mathematical notation. In fact, given such stress it is surprising the Japanese students performed as poorly as they did.

Translation problems are clearly a serious world wide. However, other countries would be wise to study Japanese, and possibly Chinese, educational techniques since these appear to be the most effective currently available.

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